MATH2050C Assignment 1

Section 2.2: 6, 8ab, 9ab, 10ab, 14ab, 15ad, 17, 18; 2.3: 4. 5d.

Hand in: 2.2: 8b, 10b, 14b, 15d, 17; 2.3: 4, 5d by Jan 16, 2024.

Supplementary Problems

The following optional problems are for you to practise mathematical induction.

1. Prove Bernoulli's Inequality:

$$(1+x)^n \geq 1+nx, \quad x \geq -1, n \geq 1 .$$

2. Prove Binomial theorem: For real a, b,

$$(a+b)^n = \sum_{k=0}^{n} C_k^n a^{n-k} b^k , \quad n \geq 1 .$$

Here $C_k^n = \frac{n!}{k!(n-k)!}$ and $0! = 1$.

3. Prove the GM-AM Inequality: For non-negative $a_1, a_2, \cdots, a_n$,

$$(a_1 a_2 \cdots a_n)^{1/n} \leq \frac{1}{n}(a_1 + a_2 + \cdots + a_n) , \quad n \geq 1,$$

and equality in the inequality holds iff all a_j's are equal.

See next page for a note on real numbers.

The Real Number System

In this course the real number system is understood to consist of four components:

1. The set of real numbers, $\mathbb{R}$, contains all rational numbers $\mathbb{Q}$.
2. On $\mathbb{R}$ there are two algebraic operations, that is, addition and multiplication.
3. There is an order structure " $<$ " on $\mathbb{R}$. For any two real numbers a, b, exactly one of the following cases holds: $a < b$, $b < a$, or $a = b$. $\mathbb{R}$ can be divided in three classes: positive numbers, negative numbers and the zero.
4. (Order Completeness Axiom) For any nonempty subset S of $\mathbb{R}$ which is bounded from above, its supremum exists in $\mathbb{R}$ (not necessarily in S). It implies that for any nonempty subset S of $\mathbb{R}$ which is bounded from below, its infimum exists in $\mathbb{R}$ (not necessarily in S). Note that the supremum/infimum may not belong to S itself.

The definitions of the supremum and infimum can be found in 2.3, Text.